NOTATION

G_{gj}, flow rate of "fresh" drying agent into j-th zone; G_{gji}, flow rate of "exhausted" drying agent of i-th zone into j-th zone; c_{1j}, c_{2ji}, c_{3ji}, constants used to "weight" different kinds of expenditures; w, moisture content of material; k, drying coefficient; τ, time; t_{wet}, t_m, t_{1m}, t_t, t_{1t}, t_{2g}, temperatures of wet-bulb thermometer, of material along length of and at entrance to dryer, of transporting devices along length of and at entrance to dryer, and of "exhausted" drying agent; G_a, flow rate of cold air; d_2, moisture content of "exhausted" drying agent; n, number of zones in dryer; x, x_{j-1}, x_j, coordinates of current, initial, and end points of j-th zone of dryer; α_1-α_{32}, constants; B, coefficient depending on mass composition of material; R_U, hydraulic radius; Δt, drying potential; v, velocity of drying agent; F, cross-sectional area of dryer; w_{cr}, critical moisture content of material.

LITERATURE CITED

1. I. P. Baumshtein, "Development of algorithms for the optimum control of drying apparatus," Summaries of Reports to All-Union Conference "Intensification of Drying Processes and the Use of New Technology for These Purposes" [in Russian], Vses. Sov. Nauchno-Inzh.-Tekh. Obshch., Moscow (1977).
2. A. M. Tsirlin, V. S. Balakirev, and E. G. Dudnikov, Variation Methods of Optimizing Controlled Objects [in Russian], Énergiya, Moscow (1976).
3. Handbook "Construction Ceramics" [in Russian], E. L. Rokhvarger (ed.), Stroiizdat, Moscow (1976).

WAVE REGIME OF FILTRATION OF SUSPENSIONS

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It is shown that a wave regime can exist for models of filtration of one-component and binary suspensions which take into account the formation of a deposit in stagnant and flowing regions of the porous bed. Theoretical formulas for the concentration in the solution and deposit phases are obtained.

1. Filtration of suspensions in a porous medium in order to purify solutions or extract components of the disperse phase is a common mass-transfer process. If clarification is effected by adhesion throughout the porous bed without formation of a film on the filter surface and at a constant filtration rate, engineering methods of calculating the time of protective action t_{pr} of the filter and the head (H) loss time are based on two empirical relations [1]:

$$t_{pr} = kl - \tau, \tag{1}$$

$$t_h = \frac{H - h_0}{m}, \tag{2}$$

which are applicable for sufficiently long beds and sufficiently long filtering cycles. A method of experimental determination of the constants contained in Eqs. (1) and (2) has been developed by technological simulation of the actual process [1].

Equation (1) was first proposed by Shilov [2] for calculation of sorption filters. It was subsequently shown in sorption theory that this equation is valid for a wave regime when a concentration front moving through the bed at constant velocity $n = k^{-1}$ is formed, and that a sufficient condition for occurrence of the wave regime is convexity of the sorption isotherm [3]. In filtration dynamics, however, the question of the existence of a wave regime has not been investigated. In addition, models [4, 5] which generally have no wave regime are widely used.

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We will establish sufficient conditions for the existence of a filtration wave regime, which will enable us to relate the empirical constants in Eqs. (1) and (2) to the physical parameters of the model and on this basis to propose more rational methods of engineering calculations of filters, and also to make more objective assessments of different filtering materials.

2. The mathematical description of filtration of suspensions in a porous medium is based on experimentally investigated features of mass transfer of particles of the disperse phase to the bed surface (filtration kinetics) and the properties of the deposit (filtration statics). It has been shown [6, 7] that there are two types of deposit — washed-out (type A) and not-washed-out (type B). The type A deposit is usually formed on the frontal parts of the grains washed by the jet component of the flow, while type B is formed mainly in stagnant zones where a circulation flow occurs [8, 9]. The ratio of the capacities of these zones is characterized by the parameter $\gamma = \rho_{a0}/\rho_{p0}$; it has been shown experimentally [6, 7, 10] that $\gamma \ll 1$. The kinetics is of the external-diffusion type. The ratio of the kinetic "activities" of the filter zones is characterized by the parameter $b = \beta_a/\beta_p$. According to experiments [7], $b \gg 1$. For many suspensions it is also essential to take into account dynamic processes in the deposit, which is characterized by the model by the deposit aging parameter q_0.

A model based on consideration of the above properties of the filtration process was proposed for one-component suspensions in [7]. We write it in dimensionless variables

$$\frac{\partial u}{\partial X} + \gamma \frac{\partial q_a}{\partial T} + \frac{\partial q_p}{\partial T} = 0, \tag{3}$$

$$\gamma \frac{\partial q_a}{\partial T} = b(u - q_a), \tag{4}$$

$$\frac{\partial q_p}{\partial T} = \alpha_0(q_p, q_0)\, u, \tag{5}$$

where $u = c/c_0$, $q_a = \rho_a/\rho_{a0}$, $q_p = \rho_p/\rho_{p0}$, $X = \beta_p x/v$, and $T = \beta_p t c_0/\rho_{p0}$. Equation (3) represents the balance of matter of the disperse phase, (4) represents the kinetics of formation of the type A deposit, and (5) is the same for the type B deposit. The isotherm of the type B deposit is rectangular, and for the usual cases encountered in practice — filtration of low-concentration suspensions — the isotherm of the type A deposit is assumed to be linear. The aging of the deposit is taken into account by the function $\alpha(q_p, q_0)$:

$$\alpha = \{1 \text{ for } 0 \leqslant q_p < q_0;\ q_0/q_p \text{ for } q_0 \leqslant q_p < 1;\ 0 \text{ for } q_p = 1\}. \tag{6}$$

We note that published methods are special cases of system (3)-(5). Thus, in the linear model of [4, 5] it is assumed that $q_p = 0$. On the other hand, in the models of [11, 12], it is assumed that $q_a = 0$. These simplifications not only distort the physical picture of the process, but also lead to erroneous conclusions. We will show that when the two types of deposits are considered the filtration model can have a wave solution when the concentrations u, q_a, and q_p are functions of the wave variable $\xi = X - \sigma T$, where σ is the dimensionless wave velocity. The solution must satisfy asymptotically the following conditions:

$$u(-\infty) = q_a(-\infty) = q_p(-\infty) = 1,\ u(\infty) = q_a(\infty) = q_p(\infty) = 0. \tag{7}$$

On conversion to wave variables in system (3)-(5) we obtain the following system of differential equations:

$$\dot{u} - \sigma \dot{q}_p - \sigma\gamma \dot{q}_a = 0, \tag{8}$$

$$-\sigma\gamma \dot{q}_a = b(u - q_a), \tag{9}$$

$$-\sigma \dot{q}_p = \alpha_0(q_p, q_a)\, u. \tag{10}$$

Integrating (8) and taking (7) into account we obtain the balance relation, which is satisfied along the whole wave,

$$u = \sigma(q_p + \gamma q_a), \tag{11}$$

and an expression for the dimensionless wave velocity: $\sigma = (1 + \gamma)^{-1}$.

We eliminate $u(\xi)$ from Eqs. (9) and (10) and, using (11), we seek the solution in the phase plane (q_p, q_a):

$$\frac{dq_a}{dq_p} = \frac{b(q_p - q_a)}{\gamma(q_p + \gamma q_a)} \text{ when } 0 \leqslant q_p < q_0, \tag{12}$$

$$\frac{dq_a}{dq_p} = \frac{b(q_p - q_a)q_p}{\gamma q_0(q_p + \gamma q_a)} \text{ when } q_0 \leqslant q_p < 1. \tag{13}$$

In Eq. (12) the point o (Fig. 1) is a saddle point [13], since the characteristic equation

$$\lambda^2 + (\gamma - b)\lambda - (b\gamma + b\gamma^2) = 0$$

can have two real roots. Hence, Eq. (12) has two integral curves emerging from the origin of coordinates

$$q_a = \varkappa_i q_p, \ \varkappa_i = \frac{-(\gamma + b) \pm \sqrt{(\gamma + b)^2 + 4\gamma^2 b}}{2\gamma^2},$$

where $\varkappa_1 > 0$ and $\varkappa_2 < 0$. Since q_a and q_p must be negative, only one curve ($\varkappa_1 > 0$) is physically valid (OA in Fig. 1). Taking into account that $\gamma/b \ll 1$, we write this solution in the form

$$q_a = (1 - \gamma/b)\, q_p \text{ for } 0 \leqslant q_p < q_0, \tag{14}$$

neglecting terms of the order $\gamma^2 b^{-1}$ and higher.

We linearize Eq. (13) by replacing γq_a in the denominator by γq_p:

$$\frac{dq_a}{dq_p} = \frac{b}{\gamma(1 + \gamma)q_0}(q_p - q_a). \tag{13a}$$

The general solution of this equation

$$q_a = K \exp[-bq_p \gamma^{-1}(1 + \gamma)^{-1} q_0^{-1}] + q_p - \gamma(1 + \gamma)q_0 b^{-1},$$

where K is a constant of integration, can be simplified by dropping the first term, in which the index of the exponent is a large negative number, and $q_p \geqslant q_0$. Hence, in the considered approximation the integral curve for the interval $q_0 \leqslant q_p < 1$ is a straight line (AB in Fig. 1)

$$q_a = q_p - \gamma q_0 b^{-1} \text{ for } q_0 \leqslant q_p < 1, \tag{15}$$

The solutions are joined at the point $A(q_0, q_0 - q_0\gamma b^{-1})$.

Thus, the linearization of Eq. (13) lies within the limits of the selected accuracy. The straight line (15) intersects the vertical $q_p = 1$ at point B $(1, 1 - \gamma q_0 b^{-1})$ and, hence, the third portion of the integral curve coincides with segment BC (Fig. 1). We determine the explicit form of the wave solution. Substituting (14) in (10), we obtain

$$q_p = q_0 \exp(-1 - \gamma)(\xi - \xi_0) \quad \text{for } \xi \geqslant \xi_0, \tag{16}$$

where the constant of integration ξ_0 is chosen to satisfy the condition $q_p(\xi_0) = q_0$, while function $q_a(\xi)$ is determined from (14).

When $\xi < \xi_0$, i.e., for $q_p > q_0$, the solution is found by substituting (15) in (10). Joining the solutions at point $\xi = \xi_0$, we obtain, within the limits of the selected accuracy

$$q_p = -q_0(1 + \gamma)[\xi - \xi_0 - (1 + \gamma)^{-1}] \quad \text{for } \xi_1 \leqslant \xi < \xi_0, \tag{17}$$

where ξ_1 is determined from the condition that when $\xi = \xi_1$ the accumulation of type B deposit ceases, i.e., $q_p(\xi_1) = 1$:

$$\xi_1 = \xi_0 + (1 + \gamma)^{-1} - q_0^{-1}(1 + \gamma)^{-1}.$$

The concentration of type A deposit in this case is

$$q_a(\xi_1) = 1 - \gamma q_0 b^{-1}. \tag{18}$$

From Eq. (9), taking into account the joining from (18), we can find the solution for $\xi < \xi_1$:

$$q_p = 1, \ q_a = 1 - \gamma q_0 b^{-1} \exp(\xi - \xi_1) \quad \text{for } \xi < \xi_1. \tag{19}$$

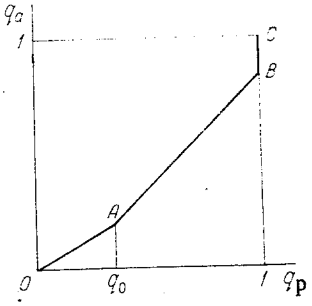

Fig. 1. Phase trajectory of concentration wave in case of filtration of one-component suspensions.

The value of ξ_0 in this case is calculated on the basis of the law of conservation of matter

$$\lim_{t_0\to\infty}\left\{\int_0^\infty\left[\rho_{a0}q_a\left(\frac{\beta_p x}{v}-\sigma\frac{\beta_p t_0 c_0}{\rho_{p_0}}\right)+\rho_{p0}q_p\left(\frac{\beta_p x}{v}-\sigma\frac{\beta_p t_0 c_0}{\rho_{p_0}}\right)\right]dx=t_0c_0v.\right.$$

This integral is easily solved in quadratures by using (16), (17), and (19).

The obtained wave solution is asymptotic for the boundary-value problem of clarification dynamics, defined by the conditions

$$u(0,\ T)=1,\ q_a(X,\ 0)=q_p(X,\ 0)=0. \tag{20}$$

The time of formation of the front can be determined from the solution of the boundary-value problem (3)-(5) and (20), and with accuracy γ it is equal to [7]: $T_{02} = 0.5(q_0 + q_0^{-1})$. For $T \geqslant T_{02}$ we can use the asymptotic solution (16), (17), and (19) and then, as was shown earlier in [14], relations (1) and (2) become valid.

We note that a wave region occurs if the type A deposit is formed in correspondence with a linear isotherm, although we know from sorption dynamics [15] that there is no wave solution in the case of a linear isotherm. The physical explanation of this is that the deposit consists mainly of type B and the wave of type B deposit "draws" the wave of type A deposit behind it. Hence, if we postulate that only type A deposit is formed ($q_p \equiv 0$), as in the models in [4, 5], the use of relations (1) and (2) is incorrect.

3. We consider the conditions of existence of the wave regime of filtration of suspensions of more complex composition. Real suspensions usually contain many components, and the fractions may differ in geometric, physical, electrokinetic, and other properties. The presently used prefiltration methods of physicochemical treatment of suspensions (coagulation, flocculation, settling) allow us to consider them in a first approximation as binary mixtures (e.g., mineral and organic fractions), and the treatment conditions usually provide more congenial filtration conditions for only one fraction — the dominant one. Hence, we will assume further that in the binary suspension the first fraction has a higher concentration and better adhesive properties. The mathematical model of clarification of a binary suspension is a generalization of system (3)-(5)

$$\frac{\partial u^{(i)}}{\partial X}+\mu_i\frac{\partial q_a^{(i)}}{\partial T}+\frac{\partial q_p^{(i)}}{\partial T}=0, \tag{21}$$

$$\mu_i\frac{\partial q_a^{(i)}}{\partial T}=b_a^{(i)}u^{(i)}-\varepsilon_i q_a^{(i)}], \tag{22}$$

$$\frac{\partial q_p^{(i)}}{\partial T}=b_p^{(i)}\alpha_i(q_p^{(1)},\ q_p^{(2)})\,u^{(i)}, \tag{23}$$

where $X = \beta_p^{(2)}x/v$; $T = \beta_p^{(2)}tc_0/\rho_{p0}$; $u(i)=c(i)/c_0$; $q_a(i) = \rho_a^{(i)}/c_0^{(i)}\Gamma_a^{(i)}$; $q_p^{(i)} = \rho_p^{(i)}/\rho_{p0}$; $\varepsilon_i = c_0^{(i)}/c_0$; $\nu_i = \rho_{p0}^{(i)}/\rho_{p0}$; $\mu_i=c_0^{(i)}\Gamma_a^{(i)}/\rho_{p0}$; $b_a^{(i)} = \beta_a^{(i)}/\beta_p^{(2)}$; $b_p^{(i)} = \beta_p^{(i)}/\beta_p^{(2)}$; $\Gamma_a^{(i)} = \rho_{a0}^{(i)}/c_0^{(i)}$; $\mu_1 + \mu_2 = \gamma$, $\varepsilon_1 + \varepsilon_2 = 1$, $b_p^{(2)} = 1$.

The above-indicated properties of a binary suspension enable us to obtain a more accurate form of the kinetic equations (22)-(23). We assume that the total capacity of the filter is determined by the first fraction, i.e., $\nu_1 = 1$, and the partial capacity of the second fraction $\nu_2 < 1$. The aging functions α_i must take into account the mutual effect of the fractions and in the general form can be represented as aging functions for one-component suspensions, but with the dependence of the aging parameters on the concentrations taken into account:

$$\alpha_1 = \begin{cases} 1 \text{ when } q_p^{(1)} < q_{01}(q_p^{(2)}) < 1 \text{ and } q_p^{(1)} + q_p^{(2)} < 1, \\ q_{01}(q_p^{(2)})/q_p^{(1)} \text{ when } q_{01}(q_p^{(2)}) \leqslant q_p^{(1)} < 1 \text{ and } q_p^{(1)} + q_p^{(2)} < 1, \\ 0 \text{ when } q_p^{(1)} = 1 \text{ or } \quad q_p^{(1)} + q_p^{(2)} = 1, \end{cases} \tag{24}$$

$$\alpha_2 = \begin{cases} 1 \text{ when } q_p^{(2)} < q_{02}(q_p^{(1)}) < \nu_2 \text{ and } q_p^{(1)} + q_p^{(2)} < 1, \\ q_{02}(q_p^{(1)})/q_p^{(2)} \text{ when } q_{02}(q_p^{(1)}) < q_p^{(2)} < \nu_2 \text{ and } q_p^{(1)} + q_p^{(2)} < 1, \\ 0 \text{ when } q_p^{(2)} = \nu_2 \text{ or } \quad q_p^{(1)} + q_p^{(2)} = 1. \end{cases} \tag{24a}$$

Thus, the deposition of the second fraction ceases either in the case of attainment of the partial capacity ν_2, or in the case of attainment of the total capacity for the sum of the fractions.

In the consideration of the possible existence of a wave solution for system (21)-(24) two cases should be noted: 1) the partial wave velocity σ_1 of the second fraction is greater than the partial wave velocity σ_2 of the first fraction: $\sigma_2 = \varepsilon/(\nu_2 + \mu_2) > (1 - \varepsilon)/(1 - \nu_2 + \mu_1) = \sigma_1$; 2) the reverse situation occurs: $\sigma_2 \leqslant \sigma_1$.

In the first case two concentration waves are formed: The wave of the second fraction moves first through the filtering bed with velocity σ_2, and is followed by the wave of the first fraction with velocity σ_1. It follows from the condition $\sigma_1 < \sigma_2$ that $\sigma_2 > (1 + \gamma)^{-1} > \sigma_1$. Thus, through a bed with capacity ρ_{po} the wave σ_2 moves more rapidly, and the wave σ_1 more slowly, than the wave of a one-component suspension with total concentration c_0. Since the wave of the second concentration can determine the time of protective action t_{pr}, it is clear that clarification of a binary suspension is less effective than that of a one-component suspension.

We consider the solution of system (21)-(24) in a two-wave variant, introducing the dependence of the concentrations on the wave variables

$$\dot{u}^{(i)} - \sigma_i(\mu_i \dot{q}_a^{(i)} + \dot{q}_p^{(i)}) = 0, \tag{25}$$

from which it follows that $u^{(i)} = \sigma_i(\mu_i q_a^{(i)} + q_p^{(i)})$. We find from Eqs. (22)-(23) that

$$-\sigma_i \mu_i \dot{q}_a^{(i)} = b_a^{(i)}[u^{(i)} - \varepsilon_i q_a^{(i)}], \tag{26}$$

$$-\sigma_i \dot{q}_p^{(i)} = b_p^{(i)} \alpha_i (q_p^{(1)}, q_p^{(2)}) u^{(i)}. \tag{27}$$

The solution of system (25)-(27) reduces to the one-component solution if the aging functions α_i can be reduced to α_0. In the two-wave variant this condition can be regarded as fulfilled in the asymptotic sense. In fact, at sufficiently large values of X and T the waves "diverge" relative to one another, so that the wave σ_1 moves through practically clean material, while wave σ_2 moves through material that is practically spent for the second fraction. Hence in Eqs. (24) and (27) we can make the aging parameters constant: $q_{02}(q_p^{(1)}) \cong q_{02}(0) = q_{02}$ and $q_{01}(q_p^{(2)}) \cong q_{01}(\nu_2) = \tilde{q}_{01}$.

System (26)-(27) can then be broken up into two independent subsystems, and the explicit form of the solution can be found by the phase-plane method, as for one-component models. We write the solution only for the initial portions of the two waves

$$q_a^{(i)} = \left(1 - \frac{\mu_i b_p^{(i)}}{\nu^{(i)} b_a^{(i)}}\right) \frac{q_p^{(i)}}{\nu^{(2)}}, \quad q_p^{(i)} = \tilde{q}_{0i} \exp \times \left[-b_p^{(i)} \left(1 + \frac{\mu_i}{\nu^{(i)}}\right)(X - \sigma_i T - \xi_{0i})\right], \tag{28}$$

where $\nu^{(1)} = 1 - \nu_2$; $\nu^{(2)} = \nu_2$ and $X - \sigma_i T \geqslant \xi_{0i}$.

It follows from solution (28), in particular, that after extraction of the main terms the concentration $q_p^{(1)}$ decreases exponentially with index $-b_p^{(1)}X$, while $q_p^{(2)}$ decreases exponentially with index $-X$. Since $b_p^{(1)} >> 1$, the expressed hypothesis of asymptotic independence of the concentration waves is valid.

We now consider the case of the ratio of partial concentrations, where $\sigma_2 \leqslant \sigma_1$ and a two-wave solution is impossible. A single wave then occurs and all the concentrations depend on the single variable $\xi = X - \sigma T$. When $\xi \to -\infty$ the conditions

$$u^{(1)} = 1 - \varepsilon,\ u^{(2)} = \varepsilon,\ q_{\mathrm{p}}^{(1)} = a,\ q_{\mathrm{p}}^{(2)} = 1 - a,\ q_{\mathrm{a}}^{(1)} = q_{\mathrm{a}}^{(2)} = 1$$

must be fulfilled.

From the balance equation (21) we derive the equation for the wave velocity: $\sigma = \varepsilon/(\gamma - \mu_1 + 1 - a) = (1 - \varepsilon)/(\mu_1 + a) = (1 + \gamma)^{-1}$. Here the equations are linearly dependent, which enables us to determine the constant $a = (1 + \gamma)(1 - \varepsilon) - \mu_1$. The wave velocity is equal to the velocity of a one-component suspension that has an initial concentration equal to the sum of the partial concentrations and is filtered through a bed with the capacity of the passive zones ρ_{po} and the active zones $\rho_{\mathrm{ao}}^{(1)} + \rho_{\mathrm{ao}}^{(2)}$.

To find the solution of system (25)-(27) in the one-wave variant we need to specify the dependence of the aging parameters on the concentrations. However, to improve the method of calculation of the filtration parameters we need only find the solutions for the initial portion of the wave, when $q_{\mathrm{p}}^{(1)} \leqslant \min\limits_{q_{\mathrm{p}}^{(2)}} q_{01}$ and $q_{\mathrm{p}}^{(2)} \leqslant \min\limits_{q_{\mathrm{p}}^{(1)}} q_{02}$. In this case the system takes the form

$$-\mu_i (1 + \gamma)^{-1} \dot{q}_{\mathrm{a}}^{(i)} = b_{\mathrm{a}}^{(i)} [u^{(i)} - \varepsilon_i q_{\mathrm{a}}^{(i)}],$$

$$-(1 + \gamma)\, \dot{q}_{\mathrm{p}}^{(i)} = b_{\mathrm{p}}^{(i)} u^{(i)}$$

and is broken up into two independent subsystems for i = 1 and i = 2, the solution of which for $u^{(i)}$ has the form

$$u^{(i)} = (1 + \gamma)^{-1} (\mu_i a_i^{-1} + 1 - \mu_i^2 a_i^{-2} b_i^{-1})\, q_{0i} \exp(-1 - \mu_i a_i^{-1})(\xi - \xi_0),$$

where $b_i = \beta_{\mathrm{a}}^{(i)}/\beta_{\mathrm{p}}^{(i)}$, $a_i = \varepsilon_i(1 + \gamma) - \mu_i$, q_{0i} is the aging parameter of the i-th fraction of the deposit.

Thus, the considered suspension filtration models can have wave solutions, which justifies the use of the fundamental relations (1) and (2) for technological calculations. Using the explicit form of the solutions it is possible to devise a method of determining the model parameters by improvement of the method of technological simulation [1].

NOTATION

c, concentration of disperse phase of suspension; c_0, concentration at filter inlet; $c^{(i)}$, $c_0^{(i)}$, corresponding concentrations of the i-th fraction; H, available head on filter; h_0, head loss on clean bed; m, rate of change of head loss in bed; n, wave velocity of concentration front; t, time; t_{pr}, time of protective action of filter; t_{h}, time available head loss; v, filtration rate; x, coordinate along filter bed; β_{a}, mass-transfer coefficient for active zones of filter; β_{p}, the same for passive (stagnant) zones; $\beta_{\mathrm{a}}^{(i)}$, $\beta_{\mathrm{p}}^{(i)}$, the same for the i-th fraction; ρ, deposit concentration; ρ_{a}, for active zones; ρ_{p}, for passive zones; ρ_{ao}, capacity of active zones; ρ_{po}, capacity of passive zones; $\rho_{\mathrm{a}}^{(i)}$, $\rho_{\mathrm{p}}^{(i)}$, the same for the i-th fraction.

LITERATURE CITED

1. D. M. Mints, Theoretical Bases of Water Purification Technology [in Russian], Stroiizdat, Moscow (1964).
2. N. A. Shilov, L. K. Lepin', and S. A. Voznesenskii, Zh. Russ. Fiz.-Khim. Ova., 61, 1107 (1929).
3. Ya. B. Zel'dovich, in: V. V. Rachinskii, Introduction to General Dynamics of Sorption and Chromatography [in Russian], Nauka, Moscow (1964).
4. D. M. Mints, Dokl. Akad. Nauk SSSR, 78, 315 (1951).
5. Yu. M. Shekhtman, Filtration of Low-Concentration Suspensions [in Russian], Izd-vo AN SSSR, Moscow (1961).
6. V. A. Alekseenko, N. G. Skvortsov, and E. V. Venitsianov, Zh. Prikl. Khim., 48, 1001 (1975).
7. E. V. Venitsianov and M. M. Senyavin, Teor. Osn. Khim. Tekhnol., 10, 584 (1976).
8. V. A. Kirillov, Yu. Sh. Matros, V. N. Sorokin, M. A. Kasamanyan, and M. G. Slin'ko, Dokl. Akad. Nauk SSSR, 206, 1409 (1972).
9. A. A. Garibyants, V. S. Golubev, and G. A. Kulikova, Izv. Akad. Nauk SSSR, Ser. Geol., No. 10 (1968).
10. B. A. Mitin, "Investigation of effect of structural-mechanical properties of deposit on operation of granular filters," Author's Abstract of Candidate's Dissertation, VODGEO, Moscow (1969).

11. K. J. Ives, J. Am. Water Works Assoc., 52, 933 (1960).
12. V. Mackrie, L'Étude du Phenomène d'Adherence, Publishing House of the Academy of Sciences of Czechoslovakia, Prague (1961).
13. V. V. Nemytskii and V. V. Stepanov, Qualitative Theory of Differential Equations [in Russian], GITTL, Moscow–Leningrad (1947).
14. E. V. Venitsianov, R. N. Rubinshtein, and M. M. Senyavin, Dokl. Akad. Nauk SSSR, 195, 658 (1970).
15. A. A. Zhukhovitskii, Ya. L. Zabezhinskii, and A. N. Tikhonov, Zh. Fiz. Khim., 19, 253 (1945).

INVESTIGATION OF NONLINEAR OSCILLATIONS OF A GAS IN OPEN PIPES

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UDC 534.222.2

Results are presented of measurements of velocity and pressure fluctuations in a pipe, open at one end, for nonlinear oscillations, excited by a piston describing harmonic motions, in the linear and nonlinear resonance regions.

It is known that nonlinear oscillations lead to a considerable intensification of various heat and mass transfer processes [1], and this subject has therefore been investigated in many papers [2-24], which have studied the oscillations occurring in pipes. In some of these [2, 3, 16-24] oscillations in closed pipes have been studied, where a piston is moving harmonically at one end, and the other end, the passive end, is closed by a diaphragm. Other papers [2-15] have examined oscillations when the passive end is open to the surrounding medium.

Periodic shock waves [3, 10, 13, 20, 24] can arise both in open and in closed pipes. The amplitude of pressure oscillations in closed pipes is proportional to the power $n = 1/2$ of the piston movement amplitude [2], while opinion varies as to the nature of the relationship in open pipes: some investigators assume $n = 1/3$ [2], while others [6, 7] take $n = 1/2$.

We note that, along with a linear resonance at frequencies

$$\omega_k = \frac{(2k-1)\pi a_0}{2L},\ k = 1, 2, 3, \ldots, \qquad (1)$$

nonlinear resonances with

$$\omega_k^* = \frac{(2k-1)\pi a_0}{4L} \qquad (2)$$

may occur in open pipes, their existence being predicted theoretically in [12, 14, 15] and verified experimentally in [13].

The great majority of authors [3, 6, 9, 13, 20] have limited their measurements to pressure oscillations, and only in [5, 7] have attempts been made to measure velocity fluctuations, but these were conducted in the comparatively slight nonlinearity region, where the fluctuations are continuous functions of time. However, it is clear that full information on the oscillations can be obtained only with simultaneous measurement of pressure and velocity.

The present paper has attempted a simultaneous investigation of pressure and velocity fluctuations in open pipes, where shock waves may originate from the open end in the frequency region of linear and nonlinear resonances.

Longitudinal oscillations of the gas column were created in a pipe with one end closed, while at the other end a plane piston was moved according to an harmonic law. In order to maximize the amplitude of oscillations, a compressor was used with a piston stroke of $2l_0 =$

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